Stabilization via semigroup interpolations

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- A parallel (and richer) theory exists on Gaussian spaces — see the monograph by Nourdin and Peccati (2012).

Framework

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- Assuming that $v(t) \geq \sigma t$, as $t \to \infty$, we want to deduce "optimal" bounds of the type

$$d_{Kol}(\widetilde{F}_t, N) := \sup_{z \in \mathbb{R}} \left| \mathbb{P}(\widetilde{F}_t \leq z) - \mathbb{P}(N \leq z) \right| \leq C\, t^{-1/2},$$

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  where $N \sim \mathcal{N}(0, 1)$.
- In a non-dynamic setting, we shall write $\eta = \eta_1$, $F = F_1$, ... and so on.

Chaos

- Recall that every $F \in L^2(\sigma(\eta))$ admits a unique chaotic decomposition of the type

$$F = \mathbb{E}(F) + \sum_{n \geq 1} I_n(f_n),$$

where

$$I_n(f_n) = \int \cdots \int f_n(x_1, ..., x_n) \mathbf{1}_{\{no\ diagonals\}} \hat{\eta}(dx_1) \cdots \hat{\eta}(dx_n)$$

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stands for a **multiple Wiener -Itô integral** with respect to the compensation $\hat{\eta} = \eta - \mathrm{Leb}$.
- This decomposition will play an important role, mostly in the background. See Günter's talk and my mini course for more details.

Type of geometric variables

One has two kind of variables: the techniques may differ very much when passing from one class to the other.

(1) Random variables having a **finite chaotic expansion**. By virtue of a result by Reitzner and Schulte (2012), these variables are basically finite linear combinations of U-statistics; examples include subgraph counting or total length statistics in the Gilbert graph.

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In this talk, we are interested in random variables of the type (2) for which the chaotic decomposition is not easily to amenable to analysis. Our idea for dealing with this situation is to suitably extend the concept of a **second-order Poincaré inequality**.

Gaussian framework

- Recall the usual **Poincaré-Chernoff-Nash inequality**: for a d-dimensional standard Gaussian vector $X = (X_1, ..., X_d)$ and for every smooth mapping $f : \mathbb{R}^d \to \mathbb{R}$,

$$\mathbf{Var}\, f(X) \leq \mathbb{E}[\|\nabla f(X)\|^2_{\mathbb{R}^d}].$$

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- The first example of a second order Poincaré estimate appears in Chatterjee (2007): for f and X as above,

$$d_{TV}(f(X), N) \leq C\, \mathbb{E}[\|\mathrm{Hess} f(X)\|^4_{op}]^{1/4} \times \mathbb{E}[\|\nabla f(X)\|^4_{\mathbb{R}^d}]^{1/4}$$

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- In Nourdin, Peccati and Reinert (2010): extension to functionals F of a general Gaussian field X,

$$d_{TV}(F, N) \leq C\, \mathbb{E}[\|D^2 F\|^4_{op}]^{1/4} \times \mathbb{E}[\|DF\|^4]^{1/4},$$

where D stands for the **Malliavin derivative.**

Towards the Poisson framework

- For a functional F of η and $x \in \mathbb{R}^d$, define $D_x F(\eta) = F(\eta + \delta_x) - F(\eta)$ (**add-one cost operator**). We shall build on the following Poincaré inequality: for every $F \in L^2(\sigma(\eta))$,

$$\mathbf{Var} F \le \mathbb{E}\left\{ \int_{\mathbb{R}^d} (D_x F)^2 dx \right\}.$$

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- In the Poisson framework, it is much easier to work with the Wasserstein distance d_W; however, the usual bound $d_{Kol} \leq 2\sqrt{d_W}$ would yield suboptimal bounds.
- One additional difficulty in the Poisson setting is that linear functionals of a Poisson measure are in general very far from being Gaussian.

Main ingredient: The Ornstein-Uhlenbeck semigroup in Mehler's form

For every $s \geq 0$, define $\eta^{(s)}$ to be a e^{-s}–thinning of η, and let $\hat{\eta}^{(s)}$ be an independent Poisson measure with intensity $(1 - e^{-s})\times$ Lebesgue. The collection of operators $\{T_s : s \geq 0\}$ given by

$$T_s F(\eta) := \mathbb{E}\left[F(\eta^{(s)} + \hat{\eta}^{(s)}) \mid \eta\right]$$

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It is sometimes convenient to work with $P_t := T_{\log 1/t}$, $t \in [0, 1]$, so that $P_0 F = \mathbb{E}(F)$ and $P_1 F = F$.

Some remarkable relations

- (Integration by parts) Consider the restriction of D to the space

$$\operatorname{dom} D := \left\{ \varphi : \mathbb{E}\left[\int \varphi(x)^2 dx \right] < \infty \right\},$$

as well as its adjoint δ. Then, for $\varphi \in \operatorname{dom}\delta$

$$\mathbb{E}[\delta(\varphi)F] = \mathbb{E} \int \varphi(x) D_x F \, dx.$$

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- Let L be the the generator of $\{T_s\}$, then $L = -\delta D$.
- The (pseudo)-inverse of L admits the representation
$$L^{-1} = -\int_0^\infty T_s \, ds = -\int_0^1 P_t \frac{dt}{t},$$
and
$$-DL^{-1} F = -\int_0^1 P_t DF \, dt.$$

Chaos representation

All these relations admit simple proofs, based on the following alternate representations. Assume $F = \sum_n I_n(f_n)$

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- $L^{-1} F = -\sum_n n^{-1} I_n(f_n)$.

A bound based on Stein's method

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A bound based on Stein's method

The following bound is due to Eichelsbacher and Thäle (2013) (building on Schulte (2012)), and is based on a subtle use of Stein's method (see my mini-course): for every $F \in L^2(\sigma(\eta))$ with mean zero and variance 1,

$$
\begin{aligned}
d_{Kol}(F, N) \leq \mathbb{E}\Big|1 - \int (D_x F)(-D_x L^{-1} F)\, dx\Big| \\
+ \frac{\sqrt{2\pi}}{8}\mathbb{E}\int (D_x F)^2 |D_x L^{-1} F|\, dx \\
+ \frac{1}{2}\mathbb{E}\int (D_x F)^2 |F||D_x L^{-1} F|\, dx \\
+ \sup_t \mathbb{E}\int (D_x \mathbf{1}\{F > t\})(D_x F)|D_x L^{-1} F|\, dx.
\end{aligned}
$$

Theorem (Last, Peccati and Schulte, 2013)

Let $F \in L^2(\sigma(\eta))$ be centered and such that **Var** $F = 1$. *Let* $N \sim \mathcal{N}(0,1)$. *then,*

$$d_{Kol}(F, N) \leq \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6,$$

or, in a dynamic setting,

$$d_{Kol}(F_t, N) \leq t \times (\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6).$$

The bounds

Here,

$$\gamma_1 := 4\sqrt{\iiint \left[\mathbb{E}(D_{x_1}F)^2(D_{x_2}F)^2\right]^{1/2}\left[\mathbb{E}(D^2_{x_1,x_3}F)^2(D^2_{x_2,x_3}F)^2\right]^{1/2} dx_1\,dx_2\,dx_3},$$

$$\gamma_2 := \left[\int \mathbb{E}(D^2_{x_1,x_3}F)^2(D^2_{x_2,x_3}F)^2\, dx_1\,dx_2\,dx_3\right]^{1/2},$$

$$\gamma_3 := \int \mathbb{E}|D_xF|^3\, dx,$$

$$\gamma_4 := \frac{1}{2}\left[\mathbb{E}F^4\right]^{1/4}\int \left[\mathbb{E}(D_xF)^4\right]^{3/4} dx,$$

$$\gamma_5 := \left[\int [\mathbb{E}(D_xF)^4\, dx\right]^{1/2},$$

$$\gamma_6 := \left[\int 6\left[[\mathbb{E}(D_{x_1}F)^4\right]^{1/2}\left[[\mathbb{E}(D^2_{x_1,x_2}F)^4\right]^{1/2} + 3[\mathbb{E}(D^2_{x_1,x_2}F)^4\, dx_1\,dx_2\right]^{1/2},$$

Wasserstein distance

One has also the simpler bound

$$d_W(F, N) \leq \gamma_1 + \gamma_2 + \gamma_3,$$

where

$$d_W(F, N) = \sup_{h:|h'|\leq 1} |\mathbb{E}[h(F)] - \mathbb{E}[h(N)]|$$

is the 1-Wasserstein distance.

Application: the nearest neighbour graph

For every t, we consider the restriction of η_t to a compact window $H \subset \mathbb{R}^d$. We build the associated k-**nearest neighbour graph** as follows: two distinct points x, y in $\eta_t \cap H$ are linked by an edge if and only if x is one of the k-nearest neighbours of y, or y is one of the k-nearest neighbours of x.

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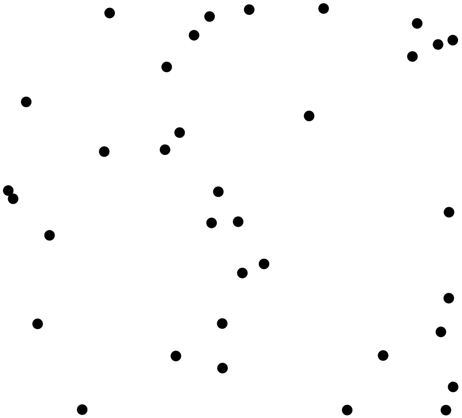
Here is an example for $k = 1$ (*courtesy of M. Schulte*)

Application: the nearest neighbour graph



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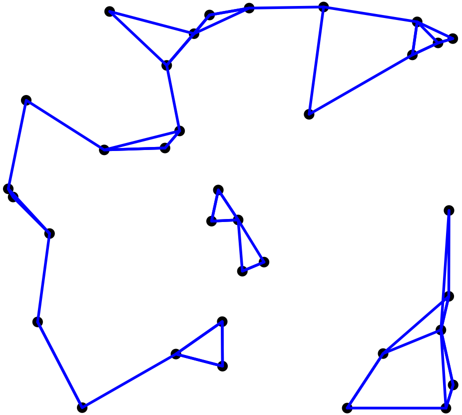

Length of the nearest neighbour graph

We wish to establish an upper bound (for $\alpha \in [0,1]$) of the type

$$d_{Kol}\left(\frac{L_t^\alpha - \mathbb{E}(L_t^\alpha)}{\mathbf{Var}^{1/2}\, L_t^\alpha}, N\right) = d_{Kol}\left(\frac{F_t - \mathbb{E}(F_t)}{\mathbf{Var}^{1/2}\, F_t}, N\right) \leq a(t),$$

where

$$L_t^\alpha := \sum_{x \sim y;\, x,y \in \eta_t \cap H} \|x - y\|^\alpha, \quad F_t = t^{\alpha/d} L_t^\alpha$$

(in such a way that $\mathbf{Var}\, F_t \geq \sigma_\alpha t$, see Penrose and Yukich (2001)).

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Previous findings for $\alpha = 1$:
Avram and Bertsimas (1993), $a(t) = O((\log t)^{1+3/4}\, t^{-1/4})$
Penrose and Yukich (2005), $a(t) = O((\log t)^{3d} t^{-1/2})$.

A general Berry-Esséen bound

Let $H \subset \mathbb{R}^d$ be a compact set.

Proposition (Last, Peccati and Schulte, 2014)

Let $F_t \in L^2(\sigma(\eta_t))$, $t \geq 1$, and assume there are finite constants $p_1, p_2, c > 0$ such that

$$\mathbb{E}|D_x F_t|^{4+p_1} \leq c, \quad \mathbb{E}|D^2_{x_1,x_2} F_t|^{4+p_2} \leq c,$$

Moreover, assume that $\mathbf{Var} F_t / t > v$, $t \geq 1$, *with* $v > 0$ *and that*

$$m := \sup_{x \in H,\ t \geq 1} t \int \mathbb{P}(D^2_{x,y} F_t \neq 0)^{p_2/(16+4p_2)}\, dy < \infty.$$

Let N be a standard Gaussian random variable. Then, there exists a finite constant C, depending uniquely on c, p_1, p_2, v, m and the measure of H, such that

$$d_{Kol}\left(\frac{F_t - \mathbb{E}(F_t)}{\sqrt{\mathbf{Var} F_t}}, N\right) \leq C\, t^{-1/2}, \quad t \geq 1.$$

Connections with stabilization theory

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- Then, it suffices to show that

$$\sup_{x,t} \int t\, \mathbb{P}\Big(y \in B^d(x, R_t(x,\eta_t)) \text{ or } R_t(x,\eta_t+\delta_y) \neq R_t(x,\eta_t)\Big)^\beta \, dy < \infty.$$

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This is very close to the **add-one cost stabilization** by Penrose and Yukich (2001).

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Further applications will appear in Günter's talk, as well as in the mini course.