Shift-Coupling and Mass-Stationarity

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Let ξ be a random measure on a locally compact second countable topological group G with left-invariant Haar measure λ. Let X be a random element taking values in a space on which G acts measurably. For instance, X could be a shift-measurable random field indexed by G. Write θ_t for the shift placing a new origin at $t \in G$.

The pair (X, ξ) is called mass-stationary if it satisfies the following (randomised) self-shift-coupling condition: for all bounded λ-continuity sets $C \subseteq G$ of positive λ-measure

$$\theta_{V_C}(X, \xi, U_C^{-1}) \stackrel{D}{=} (X, \xi, U_C^{-1})$$

where U_C is chosen independently according to $\lambda(\cdot \,|\, C)$ and V_C according to $\xi(\cdot \,|\, \theta_{U_C} C)$. Mass-stationarity is an intrinsic characterisation of Palm versions of stationary pairs.

In the talk we first motivate this definition in the case of compact groups, and also look at a randomised-background characterisation in the case $G = \mathbb{R}^d$. We then discuss a shift-coupling theory for mass-stationary pairs involving balancing transport kernels and allocations, and finally consider applications involving Cox processes and Brownian motion.